Appendix B: Non-monochromatic incident light

Monochromatic ω: $$|a_{1\to 2}(t)|^2 = \mathcal{E}_0^2 e^2 |z_{21}|^2 \frac{\sin^2\left[\frac{(E_2-E_1-\hbar\omega)t}{2\hbar}\right]}{(E_2-E_1-\hbar\omega)}$$

$$= \left(\frac{1}{2}\varepsilon_0 \mathcal{E}_0^2\right) \cdot \frac{2}{\varepsilon_0} e^2 |z_{21}|^2 \cdot \frac{\sin^2\left[\frac{(E_2-E_1-\hbar\omega)t}{2\hbar}\right]}{(E_2-E_1-\hbar\omega)}$$

$$= U_\omega \cdot \frac{2}{\varepsilon_0} e^2 |z_{21}|^2 \cdot \frac{\sin^2\left[\frac{(E_2-E_1-\hbar\omega)t}{2\hbar}\right]}{(E_2-E_1-\hbar\omega)} \quad \text{(B1)}$$

energy density

[discrete description, single ω]

How about incident light with a range of ω?

[Continuum description] $U(\omega)d\omega$ = energy density in angular frequency range $\omega \to \omega + d\omega$

$U(\omega)$ and U_ω have different units

<u>Idea</u>: Treat each frequency independently $\Rightarrow \begin{cases} |a_2(t)|^2_{\omega} \\ \vdots \\ |a_2(t)|^2_{\omega'} \end{cases}$

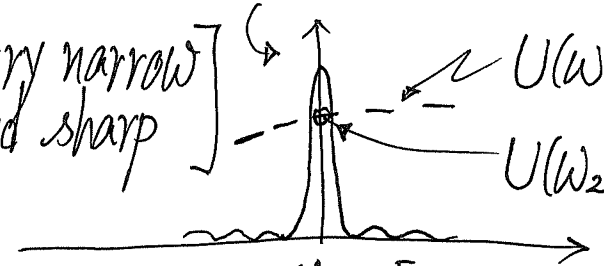
then <u>add</u> up $|a_2(t)|^2$ for different ω's

This works for <u>non-monochromatic</u> <u>incoherent</u> EM waves OR "<u>incoherent perturbations</u>"

$$|a_2(t)|^2_{\omega \to \omega + d\omega} = \frac{2}{\varepsilon_0} e^2 |z_{21}|^2 \underbrace{\overbrace{\frac{\sin^2\left[\frac{(E_2 - E_1 - \hbar\omega)}{2\hbar} t\right]}{(E_2 - E_1 - \hbar\omega)^2}}^{\text{like a delta function}}} \, U(\omega)\, d\omega \qquad \text{(B2)}$$

Area under curve $\sim t$

$$\frac{\pi}{2\hbar^2} t \underbrace{\delta\left(\frac{E_2 - E_1}{\hbar} - \omega\right)}_{\text{sharp peak at } \omega = \omega_{21}} \leftarrow \left[\text{very narrow and sharp}\right]$$



$U(\omega)$ [a smooth function across ω_{21}]

$U(\omega_{21})$

ω

$\omega_{21} \equiv \frac{E_2 - E_1}{\hbar}$

Adding up contributions from all ω's in incident light gives

$$|a_2(t)|^2 = \frac{2e^2}{\varepsilon_0}|z_{21}|^2 \underbrace{\int_0^\infty \frac{\sin^2\left[\frac{(E_2-E_1-\hbar\omega)}{2\hbar}t\right]}{(E_2-E_1-\hbar\omega)^2}}_{\text{Non-zero contribution to integral}^\dagger \text{ only at } \omega = \omega_{21} \equiv \frac{E_2-E_1}{\hbar}} U(\omega)\,d\omega$$

"the right ω"

$$= \frac{2e^2}{\varepsilon_0}|z_{21}|^2 \frac{\pi}{2\hbar^2} t\ U(\omega_{21})$$

[integrate $U(\omega)$ with $\delta(\omega_{21}-\omega)$ over ω]

pick up $U(\omega_{21})$

$$= \frac{\pi}{\varepsilon_0\hbar^2} U(\omega_{21})\, e^2|z_{21}|^2 \cdot t \qquad \text{(B3)}$$

picks up strength of light at the right ω_{21} — selection rule — linear in t

Now, $\lambda_{1\to 2} = \frac{|a_2(t)|^2}{t}$ is a quantity of unit $\left(\frac{1}{\text{time}}\right)$ (Eq.(20) on LMI-I-(40))

Ex. One can also sum over a group of states at the correct energy difference

many "states 2" at $E_2 - E_1 = \hbar\omega$

incident light

1 ——

This is often the case in solids. (energy bands of states)

Same idea: $\underbrace{D(E)\,dE}_{\text{Density of states}} = \#$ of states in interval $E \to E + dE$